Strategic Behavior and Combinatorial Betting in Prediction Markets

Yiling Chen

Yahoo! Research
New York, NY

March 26, 2008
Computer Science Meets Social Sciences

Social Sciences

Incentives
...

Information System

System Goals

Computer Science

Computational Efficiency
...

GSLIS @ UIUC
Computer Science Meets Social Sciences

Social Sciences

Incentives
... 

Ensure quality?

Computer Science

Computational Efficiency
... 

Collaborative websites?

High-quality user-contributed knowledge
Computer Science Meets Social Sciences

Social Sciences

Computer Science

Prediction Markets

Incentives

Computational Efficiency

Information aggregation

Truthfulness?

Pricing efficiently?
Outline

1. **An Overview of Prediction Markets**
2. **Strategic Behavior in Prediction Markets**
3. **Combinatorial Prediction Markets**
4. **Conclusions and Future Directions**
Q: Will Hillary Clinton win the Democratic Primary race?

Obama will win the primary race.

I bet $1000 Clinton will win the primary race.

Betting intermediaries

- Las Vegas, Wall Street, Betfair, Intrade, ...
A Prediction Market

Informal Definition

A prediction market is a betting intermediary designed to aggregate information about events of interests and generate consensus predictions.

1. Turn an uncertain event into a random variable.
   Eg. A Democrat wins 2008 Presidential election? (Y/N) $\rightarrow$ 0/1 random variable

2. Create a contract whose payoff equals value of the random variable.
   $1$ if a Democrat wins the election, $0$ otherwise

3. Open a market in the contract and attract traders to wager and speculate.
A Prediction Market

Informal Definition

A prediction market is a betting intermediary designed to aggregate information about events of interests and generate consensus predictions.

1. Turn an uncertain event into a random variable.
   Eg. A Democrat wins 2008 Presidential election? (Y/N) → 0/1 random variable

2. Create a contract whose payoff equals value of the random variable.
   $1 if a Democrat wins the election, $0 otherwise

3. Open a market in the contract and attract traders to wager and speculate.
Why Markets? – Get Information

- Speculation $\implies$ Price Discovery
  Price $\approx$ Expectation of the random variable $| \text{all information}$
  (in theory, lab experiments, and empirical studies)
Why Markets? – Get Information

speculation $\Rightarrow$ price discovery

price $\approx$ expectation of the random variable | all information
(in theory, lab experiments, and empirical studies)

$1$ if a Democrat wins the election, $0$ otherwise
Why Markets? – Get Information

- Speculation $\implies$ Price Discovery
  
  Price $\approx$ Expectation of the random variable $| \ $all information
  (in theory, lab experiments, and empirical studies)

  $\$1$ if a Democrat wins the election, $\$0$ otherwise

  Value of Contract

  ?
Speculation $\implies$ Price Discovery
Price $\approx$ Expectation of the random variable $|\text{ all information}$
(in theory, lab experiments, and empirical studies)

$1$ if a Democrat wins the election, $0$ otherwise

<table>
<thead>
<tr>
<th>Value of Contract</th>
<th>Payoff</th>
</tr>
</thead>
<tbody>
<tr>
<td>?</td>
<td>$1$</td>
</tr>
<tr>
<td></td>
<td>$0$</td>
</tr>
</tbody>
</table>
Speculation $\implies$ Price Discovery
Price $\approx$ Expectation of the random variable $|$ all information
(in theory, lab experiments, and empirical studies)

$1$ if a Democrat wins the election, $0$ otherwise

<table>
<thead>
<tr>
<th>Value of Contract</th>
<th>Payoff</th>
<th>Event Outcome</th>
</tr>
</thead>
<tbody>
<tr>
<td>?</td>
<td>$\rightarrow$ $1$</td>
<td>Democrat wins</td>
</tr>
<tr>
<td>$\rightarrow$ $0$</td>
<td></td>
<td>Democrat loses</td>
</tr>
</tbody>
</table>
Why Markets? – Get Information

- Speculation $\implies$ Price Discovery
  Price $\approx$ Expectation of the random variable $|$ all information
  (in theory, lab experiments, and empirical studies)

$1$ if a Democrat wins the election, $0$ otherwise

<table>
<thead>
<tr>
<th>Value of Contract</th>
<th>Payoff</th>
<th>Event Outcome</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P(\text{Democrat wins})$</td>
<td>$\rightarrow$ $1$</td>
<td>Democrat wins</td>
</tr>
<tr>
<td></td>
<td>$\rightarrow$ $0$</td>
<td>Democrat loses</td>
</tr>
</tbody>
</table>
Why Markets? – Get Information

- Speculation $\implies$ Price Discovery
  
  Price $\approx$ Expectation of the random variable $|\text{all information}$
  
  (in theory, lab experiments, and empirical studies)

$1$ if a Democrat wins the election, $0$ otherwise

<table>
<thead>
<tr>
<th>Value of Contract</th>
<th>Payoff</th>
<th>Event Outcome</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P(\text{Democrat wins})$</td>
<td>$\rightarrow$ $1$</td>
<td>Democrat wins</td>
</tr>
<tr>
<td></td>
<td>$\rightarrow$ $0$</td>
<td>Democrat loses</td>
</tr>
</tbody>
</table>

Equilibrium Price $\approx P(\text{Democrat wins} \mid \text{all info})$
Does It Work?

Yes, evidence from real markets, experiments, and theory

- Racetrack odds beat track experts [Figlewski 79]
- Orange Juice futures improve weather forecast [Roll 84]
- IEM beat political polls 451/596 [Forsythe 1992, 99][Oliven 95][Rietz 98][Berg 01][Pennock 02]
- HP market beat sales forecast 6/8 [Plott 00]
- Sports betting markets provide accurate forecasts of game outcomes [Gandar 98][Thaler 88][Debnath 03][Schmidt 02]
- Market games work [Servan-Schreiber 04][Pennock 01]
- Laboratory experiments confirm information aggregation [Plott 82, 88, 97] [Forsythe 90] [Chen 01]
- Theory: rational expectations [Grossman 81][Lucas 72]
- and more ...
Some Desired Properties of Prediction Markets

1. Liquidity
   - People can find counterparties to trade whenever they want.

2. Truthfulness
   - Participants reveal their information honestly and immediately.

3. Expressiveness
   - There are as few constraints as possible on the form of bets that people can use to express their opinions.

4. Computational tractability
   - The process of operating a market should be computationally manageable.
Status Quo of Prediction Market Mechanisms

1. Continuous double auction (CDA)
   - Thin market problem
   - Not truthful
   - Expressiveness $\Rightarrow$ low liquidity + high computational cost

2. Hanson’s Logarithmic Market Scoring Rule (LMSR)
   - Infinite liquidity
   - Truthfulness for myopic traders
   - Expressiveness $\Rightarrow$ high computational cost

Difficulties:
- Truthfulness
- Expressiveness and computational tractability
Our Attempts

1. Study when truthfulness is achieved in prediction markets

2. Design combinatorial prediction markets that not only provide more expressiveness but also have reasonable computational cost

Both attempts are in the framework of logarithmic market scoring rules.
An automated market maker

- Contracts
  - $1 if $o_1$
  - ... $1 if $o_n$

- Price functions
  \[ p_i(\vec{q}) = \frac{e^{q_i/b}}{\sum_{j=1}^{n} e^{q_j/b}} \]

- Uses a logarithmic scoring rule

\[ \text{Number of Outstanding Shares } q_1 \]
\[ \text{Price } p_1 \]
An automated market maker

- Contracts
  - $1 if $0_1$
  - ... $1 if $0_n$

- Price functions
  $$p_i(\vec{q}) = \frac{e^{q_i/b}}{\sum_{j=1}^{n} e^{q_j/b}}$$

- Uses a logarithmic scoring rule
An automated market maker

- Contracts
  - $1 if $o_1$
  - ... $1 if $o_n$

- Price functions
  
  \[
  p_i(\vec{q}) = \frac{e^{q_i/b}}{\sum_{j=1}^{n} e^{q_j/b}}
  \]

- Uses a logarithmic scoring rule
An Interface of LMSR

**SELECTED PREDICTION**

Barack Obama

**CURRENT PRICE**

$57.02

**TIP:** A price of $57.02 means there is currently a 57.0% chance this will occur.

Do you think:

- Chances are higher than 57.02% this will occur
- Chances are lower than 57.02% this will occur

**TIP:** A price of $57.02 means there is currently a 57.0% chance this will occur.

If you think the current odds of 57% are:

- **Way too low...**
  - **Buy 50 shares**
    - your cost $2,971.95
    - estimated new price $61.84

- **Low...**
  - **Buy 20 shares**
    - your cost $1,159.83
    - estimated new price $58.97

- **Just below...**
  - **Buy 5 shares**
    - your cost $286.30
    - estimated new price $57.51

- **Advanced...**
  - **Buy [__] shares**
    - your cost ...
    - estimated new price ...
Outline

1. An Overview of Prediction Markets
2. Strategic Behavior in Prediction Markets
3. Combinatorial Prediction Markets
4. Conclusions and Future Directions
Our Research Question

Can forward looking traders get more profit by bluffing in a LMSR?
Trader Behavior

1. Bluffing
   - Pretending to have information that one does not have
   - ... in order to mislead someone else and profit from their misinformation
   - Think poker

2. Truthful betting
   - Immediately changing market probabilities to one’s actual beliefs
General Setting

1. Two-outcome LMSR market, $\omega \in \{\omega_1, \omega_2\}$

2. Players are risk-neutral Bayesian agents with common prior

3. Each player has a private signal $s_i$ drawn from the conditional probability distribution $s_i|\omega$

4. Signal distributions are common knowledge

5. Nature selects the true outcome $\omega$ and draw the private signals at the beginning of the market

6. Players then participate in the market sequentially

7. Solution concept: Perfect Bayesian Equilibrium (PBE)
Sequence Selection Game

1. Alice and Bob are the only players in LMSR
2. Alice can select the sequence of play
3. Then, Alice and Bob each play only once according to the selected sequence.

Subgame I

\[ \begin{align*}
\omega_1 & \rightarrow p_1^0 \\
\omega_2 & \rightarrow p_2^0 \\
& \rightarrow t
\end{align*} \]

Subgame II

\[ \begin{align*}
\omega_1 & \rightarrow p_1^0 \\
\omega_2 & \rightarrow p_2^0 \\
& \rightarrow t
\end{align*} \]
Sequence Selection Game

1. Alice and Bob are the only players in LMSR
2. Alice can select the sequence of play
3. Then, Alice and Bob each play only once according to the selected sequence.

Subgame I

\[
\begin{align*}
\omega_1 & \quad p_1^0 & \quad P(\omega_1|s_A) \\
\omega_2 & \quad p_2^0 & \quad P(\omega_2|s_A)
\end{align*}
\]

Subgame II

\[
\begin{align*}
\omega_1 & \quad p_1^0 \\
\omega_2 & \quad p_2^0
\end{align*}
\]
Sequence Selection Game

1. Alice and Bob are the only players in LMSR
2. Alice can select the sequence of play
3. Then, Alice and Bob each play only once according to the selected sequence.

**Subgame I**

\[
\begin{align*}
\omega_1 & \quad p_1^0 \quad P(\omega_1 | s_A) \quad P(\omega_1 | s_A, s_B) \\
\omega_2 & \quad p_2^0 \quad P(\omega_2 | s_A) \quad P(\omega_2 | s_A, s_B)
\end{align*}
\]

**Subgame II**

\[
\begin{align*}
\omega_1 & \quad p_1^0 \\
\omega_2 & \quad p_2^0
\end{align*}
\]
Sequence Selection Game

1. Alice and Bob are the only players in LMSR
2. Alice can select the sequence of play
3. Then, Alice and Bob each play only once according to the selected sequence.

Subgame I

\[\begin{align*}
\omega_1 & \quad p_1^0 \quad P(\omega_1|s_A) \quad P(\omega_1|s_A, s_B) \\
\omega_2 & \quad p_2^0 \quad P(\omega_2|s_A) \quad P(\omega_2|s_A, s_B)
\end{align*}\]

Subgame II

\[\begin{align*}
\omega_1 & \quad p_1^0 \quad P(\omega_1|s_A, s_B) \\
\omega_2 & \quad p_2^0 \quad P(\omega_2|s_A, s_B)
\end{align*}\]
Sequence Selection Game

1. Alice and Bob are the only players in LMSR
2. Alice can select the sequence of play
3. Then, Alice and Bob each play only once according to the selected sequence.

Subgame I

\[ \begin{align*}
\omega_1 & \quad p_1^0 \quad P(\omega_1|s_A) \quad P(\omega_1|s_A, s_B) \\
\omega_2 & \quad p_2^0 \quad P(\omega_2|s_A) \quad P(\omega_2|s_A, s_B)
\end{align*} \]

Subgame II

\[ \begin{align*}
\omega_1 & \quad p_1^0 \quad ? \quad P(\omega_1|s_A, s_B) \\
\omega_2 & \quad p_2^0 \quad ? \quad P(\omega_2|s_A, s_B)
\end{align*} \]
Who Goes First

Theorem

If signals of Alice and Bob are conditionally independent then Alice selecting herself to play first is a PBE of the sequence selection game.

- For any initial market prices
- For any signal distribution and realized signal

The proof leverages information theory.
Alice and Bob are the only players in LMSR

Alice can play twice, in the first and third rounds

Bob can only play in the second round
\textbf{Alice-Bob-Alice Game}

1. Alice and Bob are the only players in LMSR
2. Alice can play twice, in the first and third rounds
3. Bob can only play in the second round

$\omega_1 \quad p_1^0 \quad p_1^1 \quad \Pr(\omega_1|p_1^1, p_1^2, s_B) \quad \Pr(\omega_1|s_A, s_B)$

$\omega_2 \quad p_2^0 \quad p_2^1 \quad \Pr(\omega_2|p_1^1, p_1^2, s_B) \quad \Pr(\omega_2|s_A, s_B)$
1 Alice and Bob are the only players in LMSR
2 Alice can play twice, in the first and third rounds
3 Bob can only play in the second round

\[
P(\omega_1 | p_1^0, p_1^1, s_B) \quad P(\omega_1 | s_A, s_B)
\]

\[
P(\omega_2 | p_2^0, p_2^1, s_B) \quad P(\omega_2 | s_A, s_B)
\]
Theorem

If signals of Alice and Bob are conditionally independent then Alice playing truthfully in the first round and Bob playing truthfully in the second round are a PBE of the Alice-Bob-Alice game.

- For any initial market prices
- For any signal distribution and realized signal
Theorem

If signals are conditionally independent, every player playing truthfully at their first chance to play is a PBE of the finite player finite round game.
Theorem

If signals are conditionally independent, every player playing truthfully at their first chance to play is a PBE of the finite player finite round game.
If signals are \textit{conditionally independent}, every player playing truthfully at their first chance to play is a PBE of the finite player finite round game.
Finite Player Finite Round Game

Theorem

If signals are conditionally independent, every player playing truthfully at their first chance to play is a PBE of the finite player finite round game.
Finite Player Finite Round Game

Theorem

If signals are conditionally independent, every player playing truthfully at their first chance to play is a PBE of the finite player finite round game.
Finite Player Finite Round Game

Theorem

If signals are conditionally independent, every player playing truthfully at their first chance to play is a PBE of the finite player finite round game.
1. Consider a Alice-Bob-Alice game
2. Alice and Bob each see an independent fair coin flip
3. Outcomes to predict: HH or not
4. There exists a bluffing equilibrium, where Alice with positive probability plays “always pretend to see H”

<table>
<thead>
<tr>
<th></th>
<th>HH</th>
<th>Other</th>
</tr>
</thead>
<tbody>
<tr>
<td>P(HH)</td>
<td>0.25</td>
<td>0.75</td>
</tr>
<tr>
<td>6P</td>
<td>0.5</td>
<td>0.5</td>
</tr>
</tbody>
</table>
Outline

1. An Overview of Prediction Markets
2. Strategic Behavior in Prediction Markets
3. Combinatorial Prediction Markets
4. Conclusions and Future Directions
Expressiveness in Getting Information

1. Things people can express today
   - A Democrat wins the election (with probability 0.55)
   - No bird flu outbreak in US before 2010
   - Yahoo’s stock price is greater than $33 by the end of 2008
   - Horse A will win the race

2. Things people can not express (very well) today
   - A Democrat wins the election if he/she wins both Florida and Ohio
   - Oil price increases & A Democrat wins & Recession in 2008
   - Yahoo’s stock price is between $34 and $36 by the end of 2008
   - Horse A beats Horse B
Expressiveness in Processing Information

1. Today's Independent Markets
   - Options at different strike prices
   - Horse race win, place, and show betting pools
   - Almost every market: Wall Street, Las Vegas, Intrade, ...

2. Events are logically related, but independent markets let traders to make the inference

3. What may be better
   - Traders focus on expressing their opinions in the way they want
   - The mechanism takes care of propagating information across logically related events
Our Research Goals

1. Design expressive betting mechanisms for both getting information and processing information.

2. Explore the tradeoff between expressiveness and computational tractability.
Combinatorics One: Boolean Logic

- \( n \) Boolean events
- Outcomes: all possible \( 2^n \) combinations of the events
  \((2^{20} = 1,048,576)\)
- Contracts:
  
  \[\text{\$1 if Boolean Formula} \quad \text{\$0 otherwise}\]

- 2-clause Boolean betting
  - A Democrat wins Florida & not Massachusetts
Restricted tournament betting

- Team A wins in round $i$
- Team A wins in round $i$ given it reaches round $i$
- Team A beats teams B given they meet.
Combinatorics Two: Permutations

- $n$ competing candidates
- Outcomes: all possible $n!$ rank orderings ($10! = 3,628,800$)
- Contracts:
  
  $1$ if Property $0$ otherwise

- Subset betting
  - Candidate A finishes at position 1, 3, or 5
  - Candidate A, B, or C finishes at position 2

- Pair betting
  - Candidate A beats candidate B.
Pricing Combinatorial Betting

LMSR price functions: \( p_i(\vec{q}) = \frac{e^{q_i/b}}{\sum_j e^{q_j/b}} \)

**Theorem**

*It is \#P-hard to price 2-clause Boolean betting, subset betting, and pair betting in LMSR.*

Reduced from \#2-SAT, calculating permanent, and counting linear extensions respectively.

**Theorem**

*It is tractable to price restricted tournament betting in LMSR.*
Pricing Tournament Betting

$1$ if team A wins in round $i$ $0$ otherwise

$1$ if team A wins in round $i$ $0$ otherwise | Teams A reaches round $i$

$1$ if team A beats team B $0$ otherwise | Teams A and B meet

\[
\begin{align*}
P(x_4=L|x_2=L) & \\
P(x_4=R|x_2=L) & \\
P(x_4=L|x_2=R) & \\
P(x_4=R|x_2=R) & \\
\end{align*}
\]

\[
\begin{align*}
x_1 & \\
x_2 & \\
x_3 & \\
x_4 & \\
x_5 & \\
x_6 & \\
x_7 & \\
x_8 & \\
x_9 & \\
x_{10} & \\
x_{11} & \\
x_{12} & \\
x_{13} & \\
x_{14} & \\
x_{15} & \\
\end{align*}
\]
Pricing Tournament Betting

$1$ if team A wins in round $i$ $0$ otherwise

$1$ if team A wins in round $i$ $0$ otherwise | Teams A reaches round $i$

$1$ if team A beats team B $0$ otherwise | Teams A and B meet

\[
P(x_4=L| x_2=L) \\
P(x_4=R| x_2=L) \\
P(x_4=L| x_2=R) \\
P(x_4=R| x_2=R)
\]
Pricing Tournament Betting

- $1$ if team A wins in round $i$ $\$0$ otherwise
- $1$ if team A wins in round $i$ $\$0$ otherwise | Teams A reaches round $i$
- $1$ if team A beats team B $\$0$ otherwise | Teams A and B meet

\[
P(x_4=L|x_2=L) \\
P(x_4=R|x_2=L) \\
P(x_4=L|x_2=R) \\
P(x_4=R|x_2=R)
\]
Outline

1. An Overview of Prediction Markets
2. Strategic Behavior in Prediction Markets
3. Combinatorial Prediction Markets
4. Conclusions and Future Directions
Conclusions

1. Strategic Behavior in LMSR
   - With conditionally independent signals, truthful betting is a PBE
   - With conditionally dependent signals, there exists distributions such that bluffing is a PBE

2. Combinatorial Betting with LMSR
   - Pricing 2-clause Boolean betting, subset betting, and pair betting is computationally hard.
   - Pricing restricted tournament betting is computationally tractable.
   - Price updates for tournament betting can be operated on a compact Bayesian network.
Future Directions

- Scientific Review System

Funding Agency/Journal  Reviewers  Scientific Network
Future Directions

- Social tagging and digital collections

Controlled Vocabulary

Social Tagging

La Promenade
Women with a parasol
Claude Monet
Impressionist
Kid
My hometown
Future Directions

Social tagging and digital collections

Controlled Vocabulary

Social Tagging

La Promenade
Women with a parasol
Claude Monet
Impressionist
Kid
My hometown

Optimal for Search
Future Directions

- Social computing
  - Prediction markets and other information aggregation mechanisms
  - User-contributed content
  - Recommender systems and reputation systems

- Economic and algorithmic aspects of the Internet
  - Internet monetization
  - Models and algorithms for the web

- Web data mining


Papers are available at http://yilingchen.net.
Logarithmic Market Scoring Rule (LMSR) [Hanson 03, 06]

An automated market maker.

- Contracts
  
  $1$ if $o_1$  ...  $1$ if $o_n$

- Cost function
  
  $$C(\vec{q}) = b \log \sum_{j=1}^{n} e^{q_j/b}$$

- Price functions
  
  $$p_i(\vec{q}) = \frac{e^{q_i/b}}{\sum_{j=1}^{n} e^{q_j/b}}$$

- A trader who changes the outstanding shares from $\vec{q}_{old}$ to $\vec{q}_{new}$ pays $C(\vec{q}_{old}) - C(\vec{q}_{new})$. 

![Graph of the cost function](image-url)
Logarithmic Market Scoring Rule (LMSR) [Hanson 03, 06]

An automated market maker.

- **Contracts**
  
  \$1 if $o_1$ \ldots \$1 if $o_n$

- **Cost function**
  \[ C(\vec{q}) = b \log \sum_{j=1}^{n} e^{q_j / b} \]

- **Price functions**
  \[ p_i(\vec{q}) = \frac{e^{q_i / b}}{\sum_{j=1}^{n} e^{q_j / b}} \]

- A trader who changes the outstanding shares from $\vec{q}_{old}$ to $\vec{q}_{new}$ pays $C(\vec{q}_{old}) - C(\vec{q}_{new})$. 

![Graph showing the cost function and price functions with a red shaded area indicating the difference between $C(100, 0)$ and $C(0, 0)$.]
An automated market maker.

- Contracts
  - $1 if $o_1$
  - $1 if $o_n$

- Cost function
  \[
  C(\vec{q}) = b \log \sum_{j=1}^{n} e^{q_j/b}
  \]

- Price functions
  \[
  p_i(\vec{q}) = \frac{e^{q_i/b}}{\sum_{j=1}^{n} e^{q_j/b}}
  \]

- A trader who changes the outstanding shares from $\vec{q}_{old}$ to $\vec{q}_{new}$ pays $C(\vec{q}_{old}) - C(\vec{q}_{new})$. 
Proper Scoring Rules

1. A set of reward functions that are used to elicit expert opinion

2. An expert who reports a probability estimate $\vec{r} = (r_1, r_2, ..., r_n)$ will get payment $s_i(\vec{r})$ if outcome $i$ happens.

3. Logarithmic scoring rule

$$s_i(\vec{r}) = a + b \log(r_i)$$

4. **Proper**: To maximize one’s expected reward, a risk-neutral agent should report truthfully.
Market Scoring Rules

1. Use a proper scoring rule, e.g. logarithmic scoring rule.
2. A trader can change the current probability estimate to a new estimate.
3. The trader pays the scoring rule payment according to the old probability estimate.
4. The trader receives the scoring rule payment according to the new probability estimate.
A LMSR Market

\[ s_i(r) = 5 \log(r_i) \]

$1$ Democrat wins

$1$ Democrat loses

0.5

GSLIS @ UIUC

41
A LMSR Market

\[ s_i(\vec{r}) = 5 \log(r_i) \]

$1$ Democrat wins

$0.5$ $0.6$ $0.8$ $0.4$ $0.9$

$1$ Democrat loses

$0.5$ $0.4$ $0.2$ $0.6$ $0.1$
A LMSR Market

$$s_i(\vec{r}) = 5 \log(r_i)$$

<table>
<thead>
<tr>
<th>$1$ Democrat wins</th>
<th>0.5</th>
<th>0.6</th>
<th>0.8</th>
<th>0.4</th>
<th>0.9</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1$ Democrat loses</td>
<td>0.5</td>
<td>0.4</td>
<td>0.2</td>
<td>0.6</td>
<td>0.1</td>
</tr>
</tbody>
</table>

Democrat wins
A LMSR Market

\[ s_i(\vec{r}) = 5 \log(r_i) \]

$1$ Democrat wins 

<table>
<thead>
<tr>
<th>Vote Result</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>0.6</td>
</tr>
<tr>
<td>0.8</td>
<td>0.4</td>
</tr>
<tr>
<td>0.9</td>
<td></td>
</tr>
</tbody>
</table>

$1$ Democrat loses

<table>
<thead>
<tr>
<th>Vote Result</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>0.4</td>
</tr>
<tr>
<td>0.2</td>
<td>0.6</td>
</tr>
<tr>
<td>0.1</td>
<td></td>
</tr>
</tbody>
</table>

Democrat wins

\[ 5 \log 0.6 \quad 5 \log 0.8 \quad 5 \log 0.4 \quad 5 \log 0.9 \]

\[ 5 \log 0.5 \quad 5 \log 0.6 \quad 5 \log 0.8 \quad 5 \log 0.4 \]
A LMSR Market

\[ s_i(r) = 5 \log(r_i) \]

<table>
<thead>
<tr>
<th></th>
<th>0.5</th>
<th>0.6</th>
<th>0.8</th>
<th>0.4</th>
<th>0.9</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1$ Democrat wins</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$1$ Democrat loses</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Democrat wins

\[ 5 \log 0.6 \quad 5 \log 0.8 \quad 5 \log 0.4 \quad 5 \log 0.9 \]

\[ 5 \log 0.5 \quad 5 \log 0.6 \quad 5 \log 0.8 \quad 5 \log 0.4 \]
A LMSR Market

$$s_i(r) = 5 \log(r_i)$$

$1$ Democrat wins

<table>
<thead>
<tr>
<th></th>
<th>0.5</th>
<th>0.6</th>
<th>0.8</th>
<th>0.4</th>
<th>0.9</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_i$ (Democrat wins)</td>
<td>0.5</td>
<td>0.4</td>
<td>0.2</td>
<td>0.6</td>
<td>0.1</td>
</tr>
</tbody>
</table>

$5 \log 0.6 - 5 \log 0.8$  $5 \log 0.4 - 5 \log 0.9$

$5 \log 0.5 - 5 \log 0.8$  $5 \log 0.8 - 5 \log 0.4$

Democrat wins
A LMSR Market

\[ s_i(\vec{r}) = 5 \log(r_i) \]

<table>
<thead>
<tr>
<th>$1$ Democrat wins</th>
<th>0.5</th>
<th>0.6</th>
<th>0.8</th>
<th>0.4</th>
<th>0.9</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1$ Democrat loses</td>
<td>0.5</td>
<td>0.4</td>
<td>0.2</td>
<td>0.6</td>
<td>0.1</td>
</tr>
</tbody>
</table>

5 log 0.9 – 5 log 0.5

5 log 0.6 – 5 log 0.8 – 5 log 0.4 – 5 log 0.9 – Democrat wins

5 log 0.5 – 5 log 0.6 – 5 log 0.8 – 5 log 0.4

5 log 0.4 – 5 log 0.2 – 5 log 0.6 – 5 log 0.1 – Democrat loses

5 log 0.5 – 5 log 0.4 – 5 log 0.2 – 5 log 0.6
A LMSR Market

\[ s_i(\bar{r}) = 5 \log(r_i) \]

\begin{align*}
\text{\$1 Democrat wins} & \quad 0.5 & 0.6 & 0.8 & 0.4 & 0.9 \\
\text{\$1 Democrat loses} & \quad 0.5 & 0.4 & 0.2 & 0.6 & 0.1
\end{align*}

5 log 0.9 − 5 log 0.5

5 log 0.9 − 5 log 0.5

5 log 0.9 − 5 log 0.5

5 log 0.1 − 5 log 0.5

Democrat wins

Democrat loses
With conditional independent signals, knowing Alice’s posterior is sufficient.

\[
P(\omega_1|s_A, s_B) = \frac{P(s_B|\omega_1)P(\omega_1|s_A)}{P(s_B|\omega_1)P(\omega_1|s_A) + P(s_B|\omega_2)P(\omega_2|s_A)}
\]
Intuition of the Alice-Bob-Alice Game

- Alice bluffing is equivalent to Alice selecting Bob as the first player in some sequence selection game

- Bluffing Alice
  \[ \text{Alice } p_{\text{bluff}} \implies \text{Bob } \implies \text{Alice} \]

- Truthful Alice
  \[ \text{Alice } p_{\text{bluff}} \implies \text{Alice } p_{\text{truthful}} \implies \text{Bob} \]